Reconstruction of a scalar-tensor theory of gravity in an accelerating universe

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The present acceleration of the Universe strongly indicated by recent observational data can be modeled in the scope of a scalar-tensor theory of gravity. We show that it is possible to determine the structure of this theory (the scalar field potential and the functional form of the scalar-gravity coupling) along with the present density of dustlike matter from the following two observable cosmological functions: the luminosity distance and the linear density perturbation in the dustlike matter component as functions of redshift. Explicit results are presented in the first order in the small inverse Brans-Dicke parameter ω^{-1}.

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Recent observational data on type Ia supernovae explosions at high redshifts $z \equiv \frac{a(t_0)}{a(t)} - 1 \sim 1$ obtained independently by two groups [1,2], as well as numerous previous arguments (see the recent reviews [3,4]), strongly support the existence of a new kind of matter in the Universe whose energy density not only is positive but also dominates the energy densities of all previously known forms of matter [here $a(t)$ is the scale factor of the Friedmann-Robertson-Walker (FRW) isotropic cosmological model, and t_0 is the present time]. This form of matter has a strongly negative pressure and remains unclustered at all scales where gravitational clustering of baryons and (non-baryonic) cold dark matter (CDM) is seen. Its gravity results in the present acceleration of the expansion of the Universe: $\ddot{a}(t_0) > 0$. In a first approximation, this kind of matter may be described by a constant Λ-term in the gravity equations as first introduced by Einstein. However, a Λ-term could also be slowly varying with time. If so, this will be soon determined from observational data. In particular, if we use the simplest model of a variable Λ-term (also called quintessence in [5]) borrowed from the inflationary scenario of the early Universe, namely an effective scalar field Φ with some self-interaction potential $U(\Phi)$ minimally coupled to gravity, then the functional form of $U(\Phi)$ can be determined from observational *cosmological functions*: either from the luminosity distance $D_L(z)$ [6,7], or from the linear density perturbation in the dustlike component of matter in the Universe $\delta_m(z)$ for a fixed comoving smoothing radius [6]. However, this model cannot account for *any* future observational data, in particular, for any functional form of $D_L(z)$. This happens because a variable Λ-term in this model should satisfy the weak-energy condition $\varepsilon_\Lambda + p_\Lambda \geq 0$. In terms of the observable quantity $H(z) \equiv \dot{a}(t)/a(t)$ describing the evolution of the expanding Universe at recent epochs, the following inequality should be satisfied [4]

$$\frac{dH^2(z)}{dz} \geq 3\Omega_{m,0} H_0^2 (1+z)^2 \ . \qquad (1)$$

Here, $H_0 = H(z=0)$ is the Hubble constant, $\Omega_{m,0}$ is the present energy density of the dustlike (CDM+baryons) matter component in terms of the critical density $\varepsilon_{\rm crit} = 3H_0^2/8\pi G$ ($c = \hbar = 1$, and an index 0 stands for the present value of the corresponding quantity). Note that the inequality (1) saturates when the Λ-term is exactly constant. It is not clear from the existing data whether (1) is satisfied at all. Actually the opposite holds: An attempt to reconstruct $U(\Phi)$ from the supernovae data [8] and fitting of existing data to a model with a linear equation of state for the Λ-term $p_\Lambda = w\varepsilon_\Lambda$, with $w < -1$ [9], shows that the possibility of violation of inequality (1), though strongly restricted, is not completely excluded. Hence it is natural and important to consider a variable Λ-term in a more general class of scalar-tensor theories of gravity where the requirement (1) does not arise. Moreover, this generalization of general relativity (GR) is inspired by present more fundamental quantum theories, like M-theory. In these theories, the scalar field Φ is just the dilaton field, hence we shall call it so below.

Thus, we are interested in a universe where gravity is described by a scalar-tensor theory, and we consider the Lagrangian density in the Jordan frame [10]

$$L = \frac{1}{2}\Big(F(\Phi)\ R - g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\Big) - U(\Phi) + L_m(g_{\mu\nu}) \ , \qquad (2)$$

where L_m describes dustlike matter and $F(\Phi) > 0$. This corresponds to the Brans-Dicke parameter $\omega = F/(dF/d\Phi)^2 > 0$. One may also introduce a function

$Z(\Phi)$ in front of the kinetic term $(\partial_\mu \Phi)^2$, but it can be set either to 1, or to -1 by a redefinition of the scalar field. Under the assumption of absence of ghosts in the theory, the second possibility requires the Brans-Dicke parameter to lie in the range $-3/2 < \omega < 0$ (see [11] for more details). Since this clearly contradicts solar system tests of GR either in the absence of $U(\Phi)$, or for $U(\Phi)$ satisfying the condition (8) below for scales of galaxies and clusters of galaxies, we will not discuss this possibility further. We do not introduce any direct coupling between Φ and L_m (though this possibility could be envisaged, too). This guarantees that the weak equivalence principle is exactly satisfied (universality of free-fall of laboratory-size objects), and also that fundamental constants, like e.g. the fine-structure constant, do not change with time in this theory. This is in very good agreement with laboratory, geophysical and cosmological data [12,13,14].

Such a scalar-tensor theory was recently considered as a model for a variable Λ-term for some special choices of $F(\Phi)$ and $U(\Phi)$ (see [15]). Our approach is just the opposite: We want to *derive* these functions from observational data. Since we have to determine *two* functions $F(\Phi)$ and $U(\Phi)$, we will need both observational functions $D_L(z)$ and $\delta_m(z)$, in contrast to GR. Then the reconstruction problem can be uniquely solved as will be shown below. Note that the angular diameter as a function of z provides the same information as $D_L(z)$ (see [4] and the second reference in [6]).

It is most appropriate for us to work in the Jordan frame (JF), in which the various physical quantities are those that are being measured in experiments, even though the Einstein frame (EF) often provides a better mathematical insight. In addition, the dilaton appears to be directly coupled to dustlike matter in the EF frame, in contrast to the JF. For a flat FRW universe with $ds^2 = -dt^2 + a^2 d\mathbf{x}^2$, the background equations in the JF are then

$$3FH^2 = \rho_m + \frac{\dot{\Phi}^2}{2} + U - 3H\dot{F} \ , \tag{3}$$

$$-2F\dot{H} = \rho_m + \dot{\Phi}^2 + \ddot{F} - H\dot{F} \ . \tag{4}$$

Their consequence is the equation for the dilaton itself:

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{dU}{d\Phi} - 3(\dot{H} + 2H^2)\ \frac{dF}{d\Phi} = 0 \ . \tag{5}$$

Combining Eqs. (3)–(4) and changing the argument from time t to redshift z, we obtain the following basic equation for $F(z)$:

$$F'' + \left[(\ln H)' - \frac{4}{1+z}\right]\ F' + \left[\frac{6}{(1+z)^2} - \frac{2(\ln H)'}{1+z}\right]\ F$$
$$= \frac{2U}{(1+z)^2 H^2} + 3\ (1+z) \left(\frac{H_0}{H}\right)^2 F_0\ \Omega_{m,0} \ , \tag{6}$$

where the prime denotes the derivative with respect to z.

The effective value of Newton's gravitational constant G_N in Eqs. (3–4) is given by the formula $G_N = 1/8\pi F$. We shall use its present value $G_{N,0}$ in the definition of the critical density $\varepsilon_{\rm crit}$. On the other hand, $G_{N,0}$ is *not* the quantity measured in laboratory Cavendish-type and solar-system experiments. For a massless dilaton, the effective gravitational constant between two test masses is given by

$$G_{\rm eff} = \frac{1}{8\pi F} \left(\frac{2F + 4(dF/d\Phi)^2}{2F + 3(dF/d\Phi)^2}\right) \ . \tag{7}$$

In our case, the dilaton is massive, so the expression (7) will be valid for physical scales R such that

$$R^{-2} \gg \max\left(\left|\frac{d^2U}{d\Phi^2}\right|, H^2, H^2\left|\frac{d^2F}{d\Phi^2}\right|\right) . \tag{8}$$

Previously, the expression $G_{\rm eff}$ was known from the post-Newtonian expansion; below we rederive it using the cosmological perturbation theory.

Let us now list the restrictions of the theory (2) which follow from solar-system and cosmological tests. The post-Newtonian parameters β and γ for this theory are:

$$\gamma = 1 - \frac{(dF/d\Phi)^2}{F + 2(dF/d\Phi)^2} \ , \tag{9}$$

$$\beta = 1 + \frac{1}{4}\ \frac{F\ (dF/d\Phi)}{2F + 3(dF/d\Phi)^2}\ \frac{d\gamma}{d\Phi} \ . \tag{10}$$

Using the upper bounds on $(\gamma - 1)$ from solar system measurements [16,17], we get

$$\omega_0^{-1} = F_0^{-1}\ (dF/d\Phi)_0^2 < 4 \times 10^{-4} \ . \tag{11}$$

So, $G_{N,0}$ and $G_{\rm eff,0}$ coincide with better than 2×10^{-4} accuracy. On the other hand, the difference between G_N and $G_{\rm eff}$ may be larger at redshifts $z \sim 1$ since neither the upper limit on β, nor the present experimental bound $|\dot{G}_{\rm eff}/G_{\rm eff}| < 6 \times 10^{-12}\ {\rm yr}^{-1}$ [17] significantly restrict $(d^2F/d\Phi^2)_0$. Note that we cannot use the nucleosynthesis bound on the change of $G_{\rm eff}$ since that time as the behavior of $G_{\rm eff}$ during the intermediate period is unknown, unless we make additional assumptions (see below).

The theory (2) describes a variable Λ-term with desired properties if the following three conditions are satisfied:

1) The Λ-term is dynamically important at present, namely, $\Omega_{\Lambda,0} \sim 0.7 \sim 2\Omega_{m,0}$, or

$$\left(\frac{\dot{\Phi}^2}{2} + U - 3H\dot{F}\right)_0 \sim 0.7\varepsilon_{\rm crit} \sim 2\rho_{m,0} \ . \tag{12}$$

2) The Λ-term has a sufficiently large negative pressure to provide acceleration of the present Universe. The condition $\ddot{a}_0 > 0$ reads:

$$2U_0 > (\rho_m + 2\dot{\Phi}^2 + 3\ddot{F} + 3H\dot{F})_0 \ . \tag{13}$$

3) The dark matter described by the Λ-term remains unclustered at scales up to $R \sim 10h^{-1}(1+z)^{-1}$ Mpc and probably even more (here $h = H_0/100$ km s^{-1} Mpc^{-1}). To achieve this, it is *sufficient* to assume that the inequality (8) is satisfied for all scales in question.

The first step of our program is purely kinematical: we determine $H(z)$ from $D_L(z)$ like in GR,

$$\frac{1}{H(z)} = \left(\frac{D_L(z)}{1+z}\right)' \ . \tag{14}$$

The functional dependence of $D_L(z)$ on the cosmological parameters, like $\Omega_{m,0}$, is of course model dependent. If $\Omega_{m,0}$ is already known from other tests, we can find already at that stage of the reconstruction a quantity such as the present effective equation of state of the dilaton from the formula (cf. [8]):

$$w_0 \equiv \frac{p_{\Lambda,0}}{\varepsilon_{\Lambda,0}} = \frac{(2/3)(d\ln H/dz)_0 - 1}{1-\Omega_{m,0}} \ . \tag{15}$$

$\varepsilon_{\Lambda,0}$ contains the term $-3H_0\dot{F}_0$, so that $\Omega_{m,0}+\Omega_{\Lambda,0} = 1$. The dilaton equation of state can be determined for $z > 0$, too; one has only to define what should be called the pressure and the energy density of the dilaton in general. Actually, we will show below that $\Omega_{m,0}$ is itself self-consistently determined from our approach, so no additional information is required to find $w(z)$.

In contrast to GR, Eq. (6) is no longer sufficient to determine $U(z)$; one should know $F(z)$, too. For this purpose we will use $\delta_m(z)$. We consider perturbations in the longitudinal gauge $ds^2 = -(1+2\phi)dt^2 + a^2(1-2\psi)d\mathbf{x}^2$. Working in Fourier space (a spatial dependence $\exp(i\mathbf{k}.\mathbf{x})$ with $k \equiv |\mathbf{k}|$ is assumed), the following equations are obtained:

$$\phi = \dot{v} = \psi - \delta F/F \ , \tag{16}$$

$$\dot{\delta}_m = -\frac{k^2}{a^2}\, v + 3\frac{d(\psi + Hv)}{dt} \ , \tag{17}$$

where the gauge invariant quantity $\delta_m \equiv (\delta\rho_m)/\rho_m + 3Hv$, and v is the peculiar velocity potential of dustlike matter. We also get

$$\begin{aligned} -3\dot{F}\dot{\phi} - \left(2\frac{k^2}{a^2}F - \dot{\Phi}^2 + 3H\dot{F}\right)\phi = \\ = \rho_m\delta_m + 3\frac{\dot{F}}{F}\,\dot{\delta F} + \left(\frac{k^2}{a^2} - 6H^2 - 3\frac{\dot{F}^2}{F^2}\right)\delta F \\ + \dot{\Phi}\dot{\delta\Phi} + 3H\dot{\Phi}\,\delta\Phi + \delta U, \end{aligned} \tag{18}$$

and the equation for the dilaton fluctuations $\delta\Phi$:

$$\begin{aligned} \ddot{\delta\Phi} + 3H\dot{\delta\Phi} + \left[\frac{k^2}{a^2} - 3(\dot{H}+2H^2)\frac{d^2F}{d\Phi^2} + \frac{d^2U}{d\Phi^2}\right]\delta\Phi = \\ = \left[\frac{k^2}{a^2}(\phi - 2\psi) - 3(\ddot{\psi} + 4H\dot{\psi} + H\dot{\phi})\right]\frac{dF}{d\Phi} \\ +(3\dot{\psi} + \dot{\phi})\dot{\Phi} - 2\phi\frac{dU}{d\Phi} \ . \end{aligned} \tag{19}$$

Let us now consider sufficiently small scales $R = 2\pi a/k$ for which the inequality (8) is well satisfied. For example, if $\delta_m(z)$ is determined from the abundance of rich clusters of galaxies, then the relevant comoving scale is $R \sim 8h^{-1}/(1+z)$ Mpc. If the r.h.s. of Eq. (8) is $\sim H^2$, then the corresponding small parameter is $R^2H_0^2 \sim 10^{-5}$. Note that we have another parameter, ω^{-1}, which is small at the present time, Eq. (11), but it need not be so small in the past. Also, this parameter may be larger than a^2H^2/k^2. For this reason, we will first keep it.

The solution of Eqs. (16–19) in the formal short-wavelength limit $k \to \infty$ can be found following the analytical method used in [6] in the GR case, confirmed numerically in [18]. The idea is that the leading terms in Eqs. (16–19) are either those containing k^2, or those with δ_m. Then, using (17) and the l.h.s. of Eq. (16), the standard form of the equation for dustlike matter density perturbation follows:

$$\ddot{\delta}_m + 2H\dot{\delta}_m + k^2a^{-2}\phi \simeq 0 \ . \tag{20}$$

Now we consider the solution of Eq. (19) of interest to us, for which $|\ddot{\delta\Phi}| \ll k^2a^{-2}|\delta\Phi|$. It corresponds to the growing adiabatic mode. So, keeping terms with k^2 in Eq. (19) and then using the r.h.s. of Eq. (16), we obtain:

$$\delta\Phi \simeq (\phi - 2\psi)\ \frac{dF}{d\Phi} \simeq -\phi\ \frac{F\ dF/d\Phi}{F + 2(dF/d\Phi)^2} \ . \tag{21}$$

In the GR case, $\delta\Phi \propto k^{-2}\phi$ in the limit $k \to \infty$, so matter producing the Λ-term is not gravitationally clustered at small scales (physically, due to free streaming). This is not so in scalar-tensor gravity: The dilaton remains partly clustered for arbitrarily small scales, this clustering being small only because ω is large.

Keeping only terms with k^2 or δ_m in Eq. (18), we get the expression of ϕ through δ_m and δF. Finally, inserting it into Eq. (20) and using Eq. (21), we arrive to the closed form of the equation for δ_m:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\,\rho_m\ \delta_m \simeq 0 \ , \tag{22}$$

with $G_{\rm eff}$ defined in (7) above. In terms of z, (22) reads:

$$\begin{aligned} H^2\ \delta_m'' + \left(\frac{(H^2)'}{2} - \frac{H^2}{1+z}\right)\delta_m' \\ \simeq \frac{3}{2}(1+z)H_0^2\frac{G_{\rm eff}(z)}{G_{N,0}}\ \Omega_{m,0}\ \delta_m \ . \end{aligned} \tag{23}$$

Eq. (22) does not contain k^2 at all. Thus, its solutions, as well as the corresponding expressions for $\delta\Phi$, do not oscillate with the frequency k/a for $k \to \infty$. This justifies the assumption about $\ddot{\delta\Phi}$ made above.

Extracting $H(z)$ (from $D_L(z)$) and $\delta_m(z)$ from observations with sufficient accuracy, we can reconstruct $G_{\rm eff}(z)/G_{N,0}$ analytically. Since, as follows from Eq. (11), the quantities $G_{\rm eff,0}$ and $G_{N,0}$ coincide with better than 0.02% accuracy, Eq. (23) taken at $z = 0$ gives

also the value of $\Omega_{m,0}$ with the same accuracy. Thus, in principle, no independent measurement of $\Omega_{m,0}$ is required.

The resulting equation $G_{\rm eff}(z) = p(z)$, where $p(z)$ is a given function following from observational data, can be transformed into a nonlinear second order differential equation for $F(z)$ if we exclude $d\Phi$ (which appears in $dF/d\Phi$) using the background equation (4), which reads

$$\Phi'^2 = -F'' - \left[(\ln H)' + \frac{2}{1+z}\right] F' + \frac{2(\ln H)'}{1+z} F - 3(1+z)\frac{H_0^2}{H^2} F_0\ \Omega_{m,0}\ . \quad (24)$$

Therefore, $F(z)$ can be determined by solving that equation provided F_0 $(= 1/8\pi G_{N,0})$ and F_0' are known.

However, this procedure can be simplified a lot under reasonable assumptions, and taking into account the small present values of $\omega^{-1} = F^{-1}(dF/d\Phi)^2$ and $\dot{G}_{\rm eff}/G_{\rm eff}$. Indeed, the value of ω^{-1} for $0 \le z \lesssim 1$ can be estimated from the first terms of its Taylor expansion $\omega_0^{-1} + z\,(d\omega^{-1}/dz)_0$. Neglecting contributions proportional to ω_0^{-1}, we then get $\omega^{-1} \sim 2\,z\,\lambda\,(d^2F/d\Phi^2)_0$, with $\lambda \equiv -(d\ln F/d\Phi)_0\,\dot{\Phi}_0/H_0$, whereas $\dot{G}_{\rm eff}/G_{\rm eff} \simeq \lambda\,H_0\,[1-(d^2F/d\Phi^2)_0]$. If $(d^2F/d\Phi^2)_0$ differs significantly from 1, we can thus conclude that $\omega^{-1} \lesssim |2\dot{G}_{\rm eff}/H_0 G_{\rm eff}| \lesssim 0.25$. On the other hand, if $(d^2F/d\Phi^2)_0$ happens to be close to 1, one can still assume that there is no special cancellation of large terms in the r.h.s. of Eq. (3), and therefore that $\dot{\Phi}_0^2 \lesssim 6F_0H_0^2$. The above estimate for ω^{-1} then gives $\omega^{-1} \lesssim 2\sqrt{6/\omega_0} \lesssim 0.1$. In both cases, we thus find that $G_{\rm eff} \simeq G_N$ in the range of z involved with better than $\sim 10\%$ accuracy. Note that the same estimate may be obtained by assuming that ω^{-1} changed *monotonically* with z and using the nucleosynthesis bound (cf. [15]). Therefore, in first approximation in ω^{-1}, $G_{\rm eff}(z) \simeq 1/8\pi F(z)$ and Eq. (23) can be used to determine $F(z)$ unambiguously. Small corrections to this result can be taken into account using perturbation theory with respect to the small parameter ω^{-1}. After $F(z)$ is found, the potential $U(z)$ is determined from Eq. (6).

Finally, using Eq. (24) we find $\Phi(z)$ by simple integration. After that, both unknown functions $F(\Phi)$ and $U(\Phi)$ are completely fixed as functions of $\Phi - \Phi_0$ in that range probed by the data. Equations (23), (6) and (24), giving the subsequent steps of the reconstruction, constitute the fundamental result of our letter.

Our results generalize those obtained in GR [6] and constrain any attempt to explain a varying Λ-term using scalar-tensor theories of gravity. Good data on $\delta_m(z)$ expected to appear soon from observations of clustering and abundance of different objects at redshifts ~ 1 and more, as well as from weak gravitational lensing, together with better data on $D_L(z)$ from more supernova events, will allow implementation of the reconstruction program and determination of the microscopic Lagrangian.

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[1] S. Perlmutter, G. Aldering, M. Della Valle *et al.*, Nature **391**, 51 (1998); S. Perlmutter, G. Aldering, G. Goldhaber *et al.*, Astroph. J., **517**, 565 (1999).

[2] P.M. Garnavich, R.P. Kirshner, P. Challis *et al.*, Astrophys. J. Lett. **493**, L53 (1998); A.G. Riess, A.V. Philipenko, P. Challis *et al.*, Astron. J. **116**, 1009 (1998).

[3] N.A. Bahcall, J.P. Ostriker, S. Perlmutter, and P.J. Steinhardt, Science, **284**, 1481 (1999).

[4] V. Sahni and A.A. Starobinsky, IJMPD, to appear (2000); astro-ph/9904398.

[5] R.R. Caldwell, R. Dave, and P.J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).

[6] A.A. Starobinsky, JETP Lett. **68**, 757 (1998); Gravit. & Cosmol. **4** (Suppl.), 88 (1998).

[7] D. Huterer and M.S. Turner, Phys. Rev. D, in press (1999) (astro-ph/9808133); T. Nakamura and T. Chiba, Mon. Not. Roy. Ast. Soc. **306**, 696 (1999).

[8] T.D. Saini, S. Raychaudhury, V. Sahni, and A.A. Starobinsky, astro-ph/9910231.

[9] R.R. Caldwell, astro-ph/9908168.

[10] P.G. Bergmann, Int. J. Theor. Phys. **1**, 25 (1968); K. Nordtvedt, Astrophys. J. **161**, 1059 (1970); R. Wagoner, Phys. Rev. D **1**, 3209 (1970).

[11] B. Boisseau, G. Esposito-Farèse, and D. Polarski, in preparation.

[12] Y. Su *et al.*, Phys. Rev. D **50**, 3614 (1994).

[13] T. Damour and F. Dyson, Nucl. Phys. B **480**, 37 (1996).

[14] A.V. Ivanchik, A.Y. Potekhin, and D.A. Varshalovich, Astron. Astroph. **343**, 439 (1999).

[15] J.P. Uzan, Phys. Rev. D **59** (1999) 123510; T. Chiba, Phys. Rev. D **60** (1999) 083508; L. Amendola, Phys. Rev. D **60** (1999) 043501; F. Perrotta, C. Baccigalupi, and S. Matarrese, astro-ph/9906066; D.J. Holden and D. Wands, gr-qc/9908026; N. Bartolo and M. Pietroni, hep-ph/9908521.

[16] T.M. Eubanks *et al.*, Bull. Am. Phys. Soc., Abstract #K 11.05 (1997).

[17] J.O. Dickey *et al.*, Science **265**, 482 (1994); J.G. Williams, X.X. Newhall, and J.O. Dickey, Phys. Rev. D **53**, 6730 (1996).

[18] C.-P. Ma, R.R. Caldwell, P. Bode, and L. Wang, Astroph. J. (Lett.) **521**, L1 (1999).